

Uncertainty estimation of gravimetric water vapor permeability determination of flat materials and finished objects



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Why uncertainty?



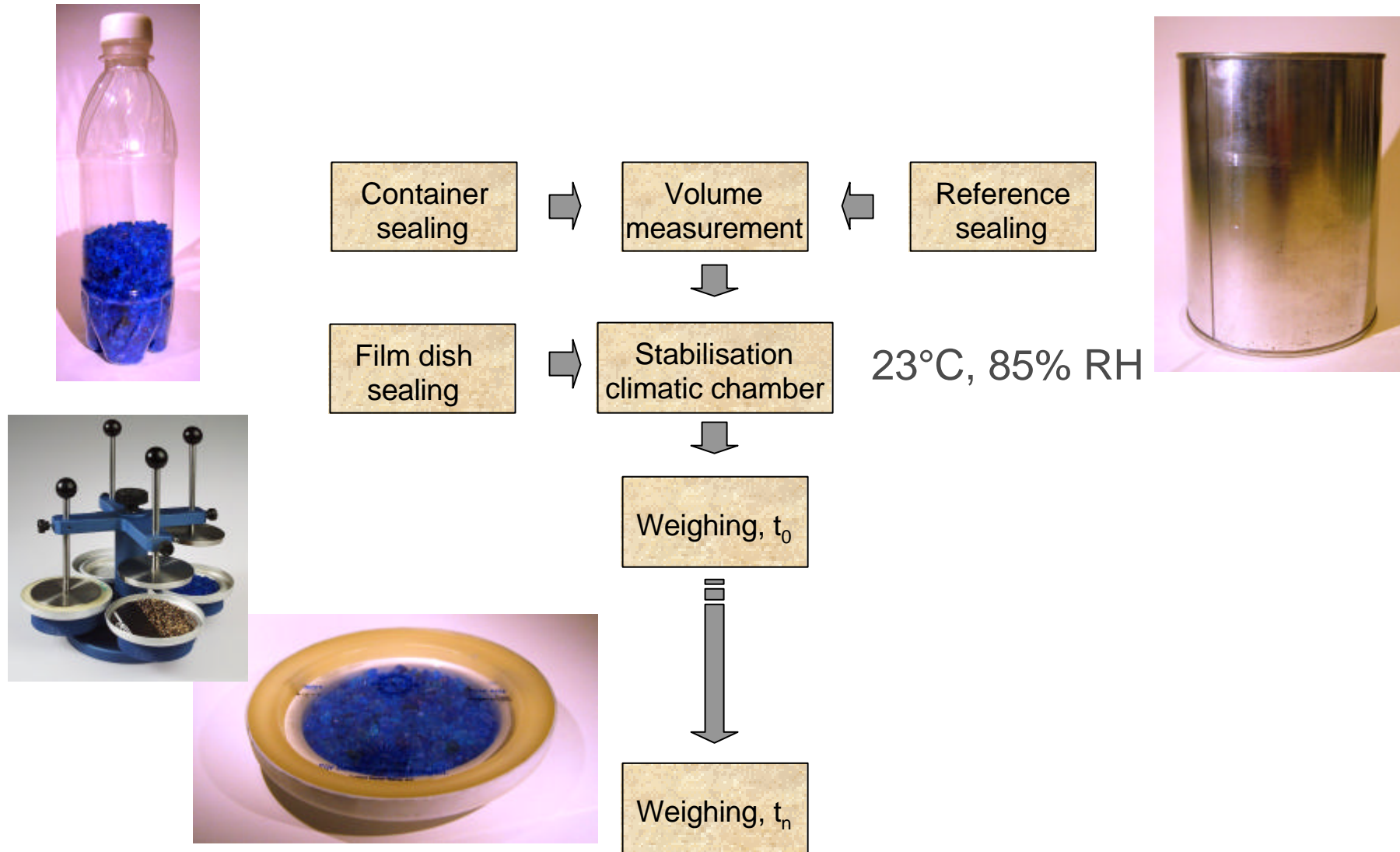
- Accredited method with widespread use within Nestlé
- Enables comparisons between laboratories
- Enables comparisons with limits
- Knowledge about uncertainty permits appropriate material choice for packaging with respect to desired shelf life
- Highlights possibilities of method improvement



Samples

Vapor barrier	Film	Finished packs
HIGH	Metallized PET (Met-PET)	Jar Co-E-P/EVOH/Co-E-P 
LOW	Pure PET (PET)	Beverage bottle PET/MXD-6/PET 

Method Flow Chart

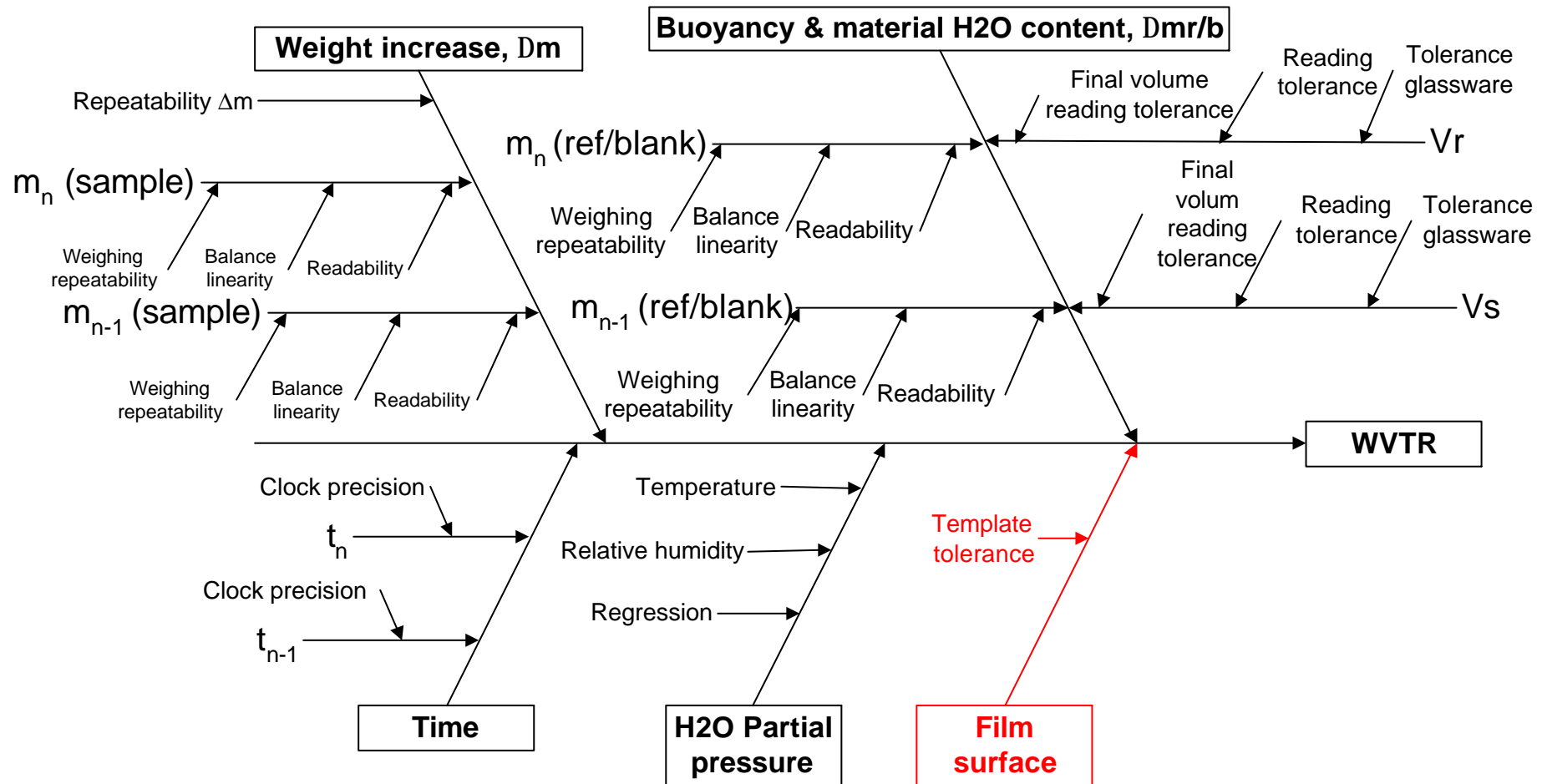


WVTR Expression

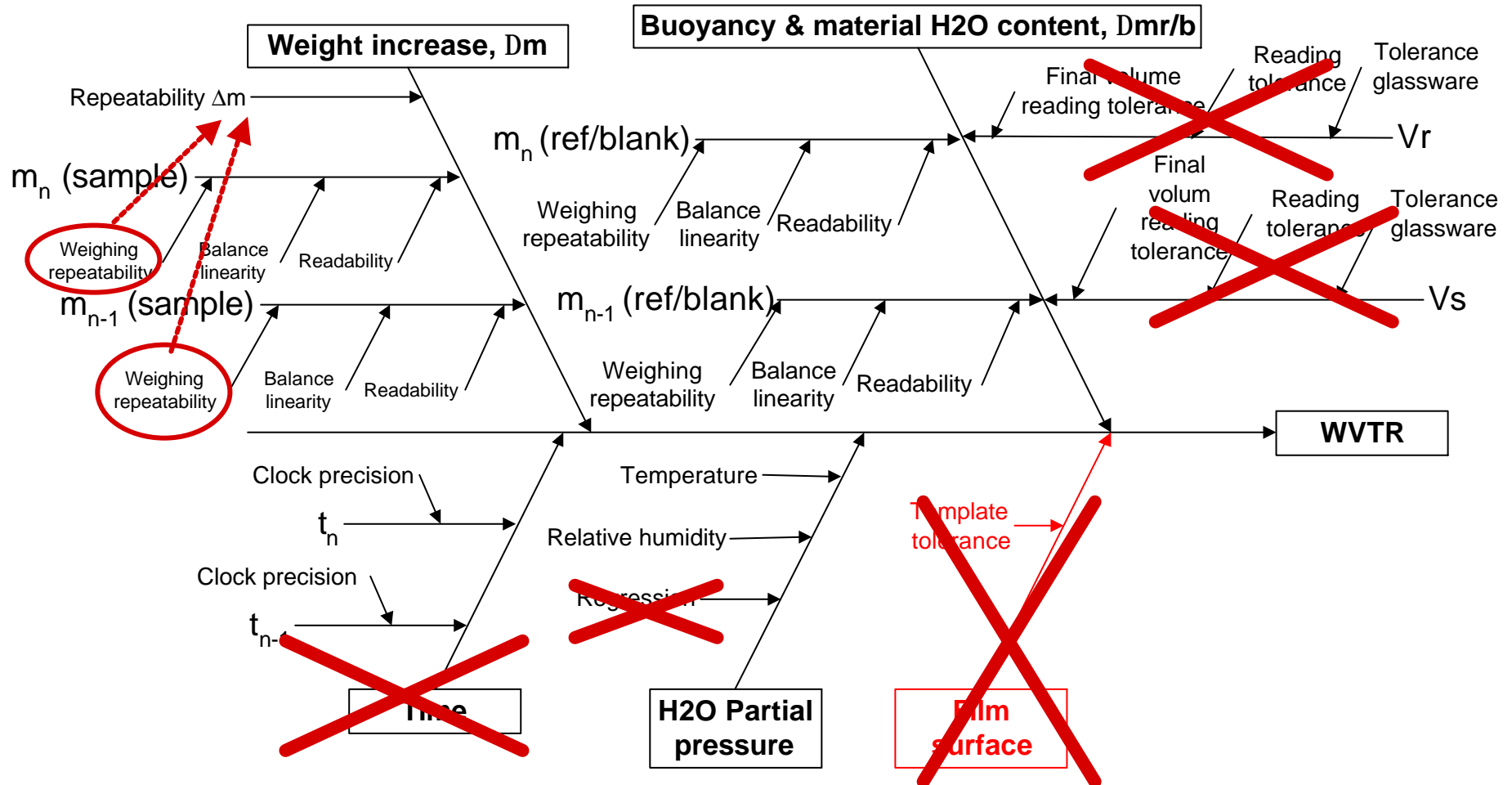
Method	Rigid container [mg/pack.day] Flexible film [g/m ² .day]
Weight gain with blank	$WVTR = \frac{(m_n - m_{n-1}) - (m_n^b - m_{n-1}^b)}{A(t_n - t_{n-1})}$
Weight gain with reference volume	$WVTR = \frac{(m_n - m_{n-1}) + (m_n^r - m_{n-1}^r)(V_s / V_r)}{(t_n - t_{n-1})}$

m = specimen mass [mg, g]
 m^b = blank weight [mg, g]
 m^r = reference weight [mg, g]
 t = time [days]
 V_s = sample volume [cm³]
 V_r = reference volume [cm³]
 A = film surface area [m²]

Cause & Effect Diagram



Refined Cause & Effect Diagram



Weight change $\Delta m / \Delta t$, $\Delta m_{b,r} / \Delta t$

$$\frac{\Delta m}{\Delta t} = \frac{(m_n - m_{n-1})}{(t_n - t_{n-1})}$$

Sample (average of several specimens)

$$u(\Delta m / \Delta t) = \frac{1}{\Delta t} \sqrt{SD_{rep}^2 + 2 * \left((u_{linearity})^2 + (u_{readability})^2 \right)}$$

Reference/blank (one specimen)

$$u(\Delta m_{r,b} / \Delta t) = \frac{1}{\Delta t} \sqrt{2 * \left((SD_{Balancerep})^2 + (u_{linearity})^2 + (u_{readability})^2 \right)}$$

H₂O Partial pressure

$$F \star (P_{\text{ext}} - P_{\text{int}})$$

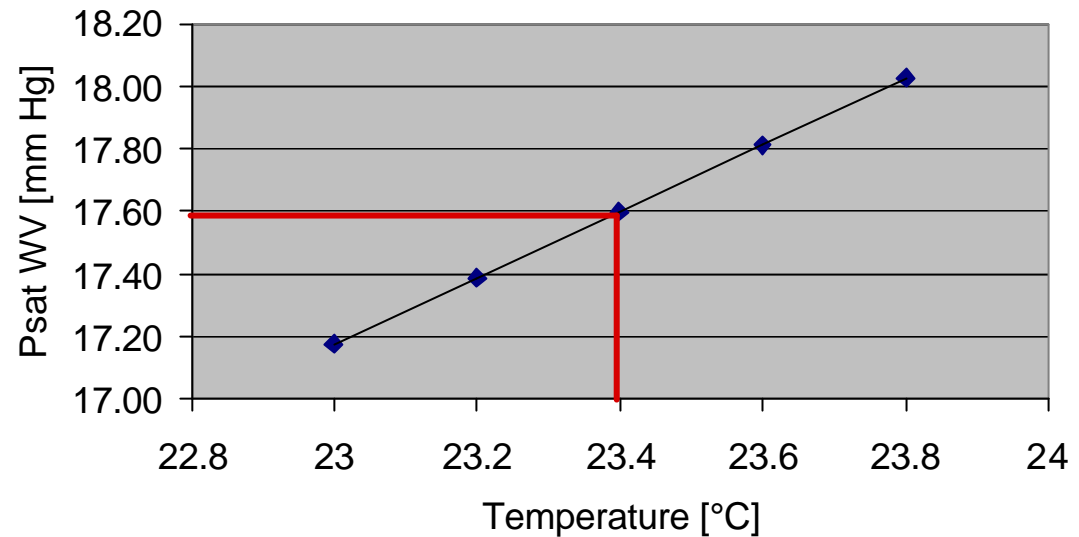
$$P_{\text{int}} = 0$$

$$P_{\text{ext}} = f(T, \text{RH})$$

$$T = 23.4 \pm 0.4 \text{ } ^\circ\text{C} = x_0$$

$$\text{RH} = 81.5 \pm 1.2 \text{ \%RH}$$

$$u_{\text{regr}} = \sqrt{SD_{\text{regr}}^2 \left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right)}$$



Source	uj	uj ²	Rel contrib.
u(P _{regr})	0.0008	7.0E-07	5.5E-06
u(P _T)	0.31	0.095	0.75
u(P _{RH})	0.18	0.032	0.25
S (u_j²)		0.127	
u_p	17.56	0.36	
Rel u_p	1	0.02	

Combined Uncertainty: Films

Met-PET

Source	Value	uj	ci	(ujci) ²	urel
$\Delta m/\Delta t$	0.0017	0.0014	199.9421	0.074	0.47
$\Delta mb/dt$	-0.0006	0.0015	-199.942	0.085	0.53
A	0.005	0.00001	-92.2913	0.000	0.00
P H ₂ O	1	0.02	0.46	0.000	0.00
Total (ciuj)²				0.160	1.00
Total (ciuj) [g/m².day]	0.46			0.40	
Relative (ciuj)				0.87	

PET

Source	Value	uj	ci	(ujci) ²	urel
$\Delta m/\Delta t$	0.04	0.0022	199.9421	0.2014	0.65
$\Delta mb/dt$	0.00	0.0015	-199.942	0.0851	0.27
A	0.005	0.00001	-1526.73	0.0005	0.00
P H ₂ O	1	0.02	7.64	0.0240	0.08
Total (ciuj)²				0.311	1.00
Total (ciuj) [g/m².day]	7.64			0.56	
Relative (ciuj)				0.07	

Combined Uncertainty: Containers & Blank



Bottle (low barrier): 21.45 ± 0.98 mg/pack.day (k=2)

Source	Value	uj	ci	(ujci) ²	urel
$\Delta m/\Delta t$	21.74	0.161	1	0.0258	0.11
$\Delta mb/dt$	0.29	0.147	-1	0.0217	0.09
P H ₂ O	1	0.02	21.45	0.1893	0.80
Total (ciuj)²				0.237	1.00
Total (ciuj) [mg/pack.day]	21.45			0.49	
Relative (ciuj)				0.02	

Jar (high barrier) 0.79 ± 0.48 mg/pack.day (k=2)

Source	Value	uj	ci	(ujci) ²	urel
$\Delta m/\Delta t$	1.21	0.190	1	0.0363	0.62
$\Delta mb/dt$	0.43	0.147	-1	0.0217	0.37
P H₂O	1	0.02	0.79	0.0003	0.00
Total (ciuj)²				0.058	1.00
Total (ciuj) [mg/pack.day]	0.79			0.24	
Relative (ciuj)				0.31	



Combined Uncertainty: Containers With Reference Volume



Bottle (low barrier): 21.85 ± 0.96 mg/pack.day (k=2)

Source	Value	uj	ci	(ujci) ²	urel
$\Delta m/\Delta t$	21.74	0.161	1	0.0258	0.11
$\Delta m_r/dt$	-0.21	0.149	-0.53	0.0061	0.03
V_s/V_r	0.53	0.026	-0.21	0.0000	0.00
P H ₂ O	1	0.02	21.85	0.1964	0.86
Total (ciuj)²				0.228	1.00
Total (ciuj) [mg/pack.day]	21.85			0.48	
Relative (ciuj)				0.02	

Jar (high barrier) 1.32 ± 0.42 mg/pack.day (k=2)

Source	Value	uj	ci	(ujci) ²	urel
$\Delta m/\Delta t$	1.21	0.190	1	0.0363	0.84
$\Delta m_r/dt$	-0.21	0.149	-0.52	0.0060	0.14
V_s/V_r	0.520	0.026	-0.21	0.0000	0.00
P H ₂ O	1	0.02	1.32	0.0007	0.02
Total (ciuj)²				0.043	1.00
Total (ciuj) [mg/pack.day]	1.32			0.21	
Relative (ciuj)				0.16	



- Present method only suited for low to medium barrier materials
- Dominant uncertainty contribution from climate control uncertainty for low barrier containers, significant contribution for low barrier films
- Sensitivity of balance limiting factor for high barrier materials
 - Prolong testing period
 - Use balance with higher sensitivity
- Difference in values between blank and reference volume measurements probably due to steady state moisture content of material in container walls

Film surface area, A

$$D = 79.8 \pm 0.2 \text{ mm}$$

$$c_D = \frac{\partial}{\partial D} \left(\frac{\rho D^2}{4} \right) = \frac{\rho D}{2}$$

A [m2]	interval	distribution	uj	ci	ciuj
0.005	0.0002	rectangular	0.000115	0.12535	1.45E-05
				Urel	0.003

Sample volume/reference volume



Uncertainty of one volume measurement

Source	interval	distribution	uj	ci	ciuj	(ciuj)^2	urel
filling	20	rectangular	11.5	1	11.5	133	0.25
reading	20	rectangular	11.5	1	11.5	133	0.25
line marking	28	rectangular	16.2	1	16.2	261	0.49
Total (ciuj)^2						528	1.00
Total (ciuj)						23.0	

Expression

$$\frac{V_s}{V_r}$$

Sensitivity coefficients (ci) for combined uncertainty

$$\frac{\partial \left(\frac{V_s}{V_r} \right)}{\partial V_s} = \frac{1}{V_r} \quad \frac{\partial \left(\frac{V_s}{V_r} \right)}{\partial V_r} = -\frac{V_s}{V_r^2}$$

Combined uncertainty beverage bottle, u(V)

Source	Value	uj	ci	(ujci)^2	urel
Vs	525	23.0	0.001	0.00053	0.78
Vr	1000	23.0	-0.000525	0.00015	0.22
Tot (ujci)^2				0.001	
Tot (ujci)	0.525			0.026	
Rel u(V)				0.049	

